



FUZZY EOQ MODELING FOR MULTIFACTOR DEPENDENT DEMAND

H. P. Umap

*Department of Statistics, Y.C. Institute of Science, Satara (415001), Maharashtra, India.
E-mail: umaphemant@gmail.com*

ARTICLE INFO

Received: 19 August 2021
Revised: 9 September 2021
Accepted: 26 September 2021
Online: 30 December 2021

To cite this paper:

H.P. Umap (2021). Fuzzy
EOQ Modeling for
Multifactor Dependent
Demand. *International Journal
of Mathematics, Statistics and
Operations Research*. 1(2): pp.
187-195

Abstract: This article formulates a fuzzy inventory model for deteriorating items in which demand is a function of selling price and advertisement frequency. The function principle method is employed for defuzzification to evaluate the total fuzzy inventory cost. The median rule is applied to determine the optimal Economic Order Quantity (EOQ) and the associated shortage quantity. The solution methodology is demonstrated through a numerical example, followed by a sensitivity analysis to assess the impact of parameter variations on the optimal solution.

Keywords: Fuzzy Inventory Model; Deteriorating Items; Price and Advertisement Dependent Demand; Economic Order Quantity (EOQ)

1. Introduction

In classical (crisp) Economic Order Quantity (EOQ) models, all inventory parameters were assumed to be fixed or could be predicted with certainty, but in real life situations, they will fluctuate little from their actual values. Hence these parameters could not be assumed as constant. Such situation can tackle through Fuzzy Set Theory by considering these parameters as fuzzy numbers. Prof. L. A. Zadeh proposed the term "FUZZY" in 1965. Prof. L. A. Zadeh [10] formally discussed the concept "Fuzzy sets". After that Zimmermann [11], Dubois and Prade [4] etc. are developed the fuzzy set approach and it is widely used in many applied subjects. Recently, some inventory models are also discussed using different type of fuzzy numbers. Vujosevic, et. al. [7] considered a modification of EOQ formula in the presence of imprecisely estimation of holding and ordering costs where costs are represented by Trapezoidal Fuzzy Number (TrFN). Yao et. al. [8, 9] used Centroid and Signed distance methods to defuzzify the total cost in without backorder inventory model, where total demand and the holding cost per unit per day are represented by triangular fuzzy numbers. Umap and Bajaj [6] also used Centroid and Signed distance methods to defuzzify the total cost. Chen S.H. and Wang C.C. [1] and Chen S.H [2] used Function principle

method to defuzzify the total cost. Umap [5] used function principle method for defuzzification where fuzzy parameters are represented by trapezoidal membership function.

Here in this article the total cost and optimum order quantity are obtained in fuzzy sense for deteriorating items and especially considering demand being dependent on selling price and frequency of advertisement by using function principal method for defuzzification to obtain total fuzzy inventory cost. Also, the median rule is applied to find the optimum Economic Order Quantity [EOQ] and shortage quantity. Solution procedure is illustrated by a numerical example. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.

2. Assumptions

1. The scheduling period is constant and no lead-time.
2. Demand rate R is dependent linearly on the unit selling price and non-linearly on frequency of advertisement i.e., $R = (a-bp)N^\alpha$ where a , b and α are non-negative constants.
3. Shortages are allowed and totally backlogged.
4. Deteriorating rate is age specific failure rate.
5. The advertisement cost is fraction of the total selling price per cycle.

3. Notations

T : Scheduling time of one cycle.

R : Demand rate per unit time; $R = (a-bp)N^\alpha$.

θ : Deterioration rate.

$Q(t)$: Inventory level at time t .

C_H : Total Holding cost per cycle.

C_1 : Holding cost per unit.

C_S : Total Shortage cost per cycle.

C_2 : Shortage cost per unit.

S_d : Total deteriorating units.

C_D : Total deteriorating cost per cycle.

C_d : Deteriorating cost per unit.

C_A : Advertisement cost per cycle.

P : Selling price per unit.

N : Number of advertisements.

μ : Advertisement cost ($0 < \mu < 1$)

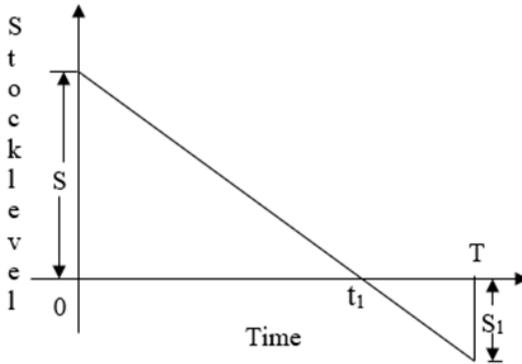
S : Initial stock level.

S_1 : Maximum shortage level.

TC : Total inventory cost per cycle.

(wavy bar (\sim) represents the fuzzification of the parameters)

4. Figure:



5. Mathematical Analysis

5.1. Crisp Model

At time $t = 0$, the initial stock level is S , as t increases the inventory level decreases due to demand mainly and partially by deterioration. The stock reaches to zero level at $t = t_1$. Then shortages occur and accumulate to the level S_1 at $t = T$.

The differential equation describing the state of inventory in the interval $(0, t_1)$ is given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = -(a - bp)N^\alpha ; 0 \leq t \leq t_1$$

Solving above differential equation using boundary condition at $t = 0, Q(t) = S$, We get ,

$$Q(t) = -\frac{(a - bp)N^\alpha}{\theta} + \left(\frac{S\theta + (a - bp)N^\alpha}{\theta}\right)e^{-\theta t} ; 0 \leq t \leq t_1$$

using boundary condition at $t = t_1, Q(t_1) = 0$, we get

$$t_1 = \frac{1}{\theta} \log \left(1 + \frac{S\theta}{(a - bp)N^\alpha}\right)$$

The differential equation describing the state of inventory in the interval (t_1, T) is given by ,

$$\frac{dQ(t)}{dt} = -(a - bp)N^\alpha \quad ; \quad t_1 \leq t \leq T$$

integrating both sides and solving using condition at $t = t_1$, $Q(t_1) = 0$, we get ,

$$Q(t) = -(a - bp)N^\alpha t + (a - bp)N^\alpha t_1 \quad ; \quad t_1 \leq t \leq T$$

using condition at $t = T$, $Q(t) = -S_1$, we get ,

$$S_1 = (a - bp)N^\alpha T - (a - bp)N^\alpha \cdot \frac{1}{\theta} \log \left(1 + \frac{S\theta}{(a - bp)N^\alpha} \right)$$

Total deteriorating units during the time interval $(0, T)$ are

$$S_d = \int_0^{t_1} \theta Q(t) dt \quad ; \quad 0 \leq t \leq t_1$$

$$S_d = -\theta \int_0^{t_1} \frac{(a - bp)N^\alpha}{\theta} + \left(\frac{S\theta + (a - bp)N^\alpha}{\theta} \right) e^{-\theta t} dt$$

Solving above integral, we get ,

$$S_d = -(a - bp)N^\alpha t_1 - \left(\frac{S\theta + (a - bp)N^\alpha}{\theta} \right) (e^{-\theta t_1} - 1)$$

Therefore the deteriorating cost is given by , $C_D = C_d S_d$

$$C_D = C_d \left[-(a - bp)N^\alpha t_1 - \left(\frac{S\theta + (a - bp)N^\alpha}{\theta} \right) (e^{-\theta t_1} - 1) \right]$$

Holding cost over the time period $(0, T)$ is given by , $C_H = C_1 \int_0^{t_1} Q(t) dt$

Solving above integral using equation (2), we get

$$C_H = C_1 \left[-\frac{(a - bp)N^\alpha t_1}{\theta} - \left(\frac{S\theta + (a - bp)N^\alpha}{\theta^2} \right) (e^{-\theta t_1} - 1) \right]$$

Shortage cost is given by

$$C_S = C_2 \left[\int_{t_1}^T -Q(t) dt \right]$$

Solving above integral by using equation (5), we get

$$C_S = C_2 \left[\frac{(a - bp)N^\alpha}{2} (T - t_1)^2 \right]$$

Advertisement cost per cycle is

$$C_A = \mu(S - S_d)PN$$

$$C_A = \mu \left[S - \frac{S^2\theta}{(a - bp)N^\alpha} \right] PN$$

Then the total inventory cost is given by ,

$$TC = C_H + C_D + C_S + C_A$$

$$TC = (C_1 + C_d \theta) \left[-\frac{(a-bp)N^\alpha t_1}{\theta} - \left(\frac{S\theta + (a-bp)N^\alpha}{\theta^2} \right) (e^{-\theta t_1} - 1) \right] + C_2 \left[\frac{(a-bp)N^\alpha}{2} (T - t_1)^2 \right] + \mu \left[S - \frac{S^2 \theta}{(a-bp)N^\alpha} \right] P.N$$

The above equation can be simplified using series form of logarithmic term and ignoring second and higher terms as follows

$$TC = (C_1 + C_d \theta) \frac{S^2}{(a-bp)N^\alpha} + C_2 \left[\frac{(a-bp)N^\alpha}{2} \left(T - \frac{S}{(a-bp)N^\alpha} \right)^2 \right] + \mu \left[S - \frac{S^2 \theta}{(a-bp)N^\alpha} \right] PN$$

To obtain optimum order quantity differentiating TIC partially w.r.t. S and equate to zero

$$\frac{dTC}{dS} = \frac{2(C_1 + C_d \theta)}{(a-bp)N^\alpha} S + \frac{C_2}{(a-bp)N^\alpha} S - \frac{2\mu PN}{(a-bp)N^\alpha} S + \mu PN - C_2 T = 0$$

The optimum order level is given by,

$$S^0 = \frac{(a-bp)N^\alpha (C_2 T - \mu PN)}{2(C_1 + C_d \theta - \mu PN \theta) + C_2}$$

6. Methodology

6.1 Function Principle method

Defining the economic order quantity (EOQ) under fuzzy inventory model requires arithmetic operations on fuzzy quantities. It appears that the method known as the function principle is more useful for the fuzzy numbers with trapezoidal membership function as shown in fig -3.1

$$\mu_A(X) = \begin{cases} \frac{w(x-c)}{(a-c)} & c \leq x \leq a \\ w & a \leq x \leq b \\ \frac{w(x-d)}{(b-d)} & b \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

Where $0 \leq w \leq 1$

Such a number will be given a brief notation (c,a,b,d;w) or even (c,a,b,d) if the maximum w is understood. In the remainder of the paper, we deal only with non-negative $c \leq a \leq b \leq d$

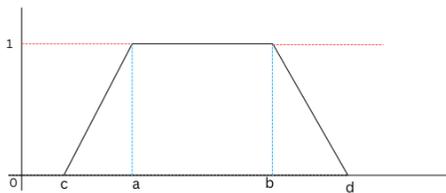


Fig 1 : Trapezoidal membership function

By using median rule to obtain find the minimization of C. The median of C_m of C can be derived from

$$[(C_m-C_1)+(C_m-C_2)]/2=[(C_4-C_m)+(C_3-C_m)] / 2.$$

Then $C_m=(C_1+C_2+C_3+C_4) / 4$

In all practical applications, C_m falls in between C_2 and C_3 . In some unlikely cases as shown in figure- it could happen that $C_m > C_3$ (or $C_m < C_2$). Here we do not consider such cases.

7. Fuzzy Model

Let the fuzzy parameters such as deterioration rate, holding cost and shortage cost are represented by trapezoidal fuzzy numbers. Then the equation of fuzzy total inventory cost is

$$T\tilde{C} = (\tilde{C}_1 \oplus C_d \otimes \tilde{\theta}) \frac{\tilde{S}^2}{(a-bp)N^\alpha} \oplus \tilde{C}_2 \left[\frac{(a-bp)N^\alpha}{2} \left(T - \frac{\tilde{S}}{(a-bp)N^\alpha} \right)^2 \right] \oplus \tilde{\mu} \otimes \left[\tilde{S} - \frac{\tilde{S}^2 \otimes \tilde{\theta}}{(a-bp)N^\alpha} \right] P.N$$

where

$$\tilde{S} = \frac{(a-bp)N^\alpha \otimes (\tilde{C}_2 \otimes T - \tilde{\mu} \otimes P.N)}{2(\tilde{C}_1 + C_d \otimes \tilde{\theta} - \tilde{\mu} \otimes \tilde{\theta} \otimes P.N) \oplus \tilde{C}_2}$$

And $\frac{(\tilde{S})^2}{(a-bp)N^\alpha}$, $\frac{\tilde{S}^2 \otimes \tilde{\theta}}{(a-bp)N^\alpha}$, $\tilde{C}_2 \otimes T$, $\tilde{\mu} \otimes P.N$, $C_d \otimes \tilde{\theta}$ and $\tilde{\mu} \otimes \tilde{\theta} \otimes P.N$ are fuzzy points

3.7.1 Defuzzification by Function Principle

Let, suppose \tilde{C}_1 , \tilde{C}_2 and $\tilde{\theta}$ are fuzzy numbers with the trapezoidal membership function. such as

$$\tilde{C}_1 = (C_{11}, C_{12}, C_{13}, C_{14})$$

$$\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$$

$$\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$$

$$\tilde{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)$$

By using function principle, the membership function of TIC can be defined as $T\tilde{C} = (TC_1, TC_2, TC_3, TC_4)$

Where

$$TC_i = \frac{(C_{1i} + C_d * \theta_i) S^2}{(a-bp)N^\alpha} + C_{2i} \left[\frac{(a-bp)N^\alpha}{2} \left(T - \frac{S}{(a-bp)N^\alpha} \right)^2 \right] + \mu_i \left[S - \frac{S^2 * \tilde{\theta}}{(a-bp)N^\alpha} \right] P.N \quad i = 1, 2, 3, 4$$

By using median rule, above equation can be revised as

$$TC_m = \frac{1}{4} \left\{ \left(\sum_{i=1}^4 C_{1i} + \sum_{i=1}^4 C_d * \theta_i \right) \frac{S^2}{(a-bp)N^\alpha} + \left(\sum_{i=1}^4 C_{2i} \right) * \left(\frac{(a-bp)N^\alpha}{2} \left[T - \frac{S}{(a-bp)N^\alpha} \right]^2 \right) + \left(\sum_{i=1}^4 \mu_i \right) * \left[S - \frac{S^2 * \tilde{\theta}}{(a-bp)N^\alpha} \right] P.N \right\}$$

To obtain optimum order quantity differentiating TIC_m partially w.r.t. S and equate to zero

$$\frac{dT C_m}{ds} = 0$$

$$\Rightarrow \frac{1}{4} \left\{ 2 * \left(\sum_{i=1}^4 C_{1i} + \sum_{i=1}^4 C_d * \theta_i \right) \frac{S}{(a-bp)N^\alpha} + \frac{\left(\sum_{i=1}^4 C_{2i} \right)}{2(a-bp)N^\alpha} * \left[((a-bp)N^{\alpha-T} - S) * (-1) \right] + \left(\sum_{i=1}^4 \mu_i \right) * \left[1 - \frac{2S * \theta_i}{(a-bp)N^\alpha} \right] P . N \right\}$$

Solving the above equation, the optimum order level is given by,

$$S^0 = \frac{(a-bp)N^\alpha * \left(\sum_{i=1}^4 C_{2i} * T - \sum_{i=1}^4 \mu_i * P . N \right)}{2 \left(\sum_{i=1}^4 C_{1i} + \sum_{i=1}^4 C_d * \theta_i - \sum_{i=1}^4 \mu_i * \theta_i * P . N \right) + \left(\sum_{i=1}^4 C_{2i} \right)}$$

8. Numerical Example

1) Crisp model :

Input : a=100, b=0.5, P=4, N=2, α =0.3, C₁=0.5, C₂=5, C_d=4, T=1, θ =0.05, μ =0.05,

Output : S=86.18, t₁=0.70, S₁=35.97, TC=81.10

2) Fuzzy Model

SENSITIVITY ANALYSIS

| N | α | C ₁ | C ₂ | θ | μ | S | t ₁ | S ₁ | TC |
|---|----------|----------------|----------------|-------------------|-------------------|-------|----------------|----------------|--------|
| 1 | 0.3 | (.3,.4,.5,.6) | (3,4,5,6) | (.01,.03,.05,.07) | (.03,.04,.05,.06) | 74.20 | 24.90 | 0.74 | 60.23 |
| | 0.3 | (.4,.5,.6,.7) | (4,5,6,7) | (.03,.05,.07,.09) | (.04,.05,.06,.07) | 73.36 | 26.24 | 0.73 | 75.83 |
| | 0.4 | (.3,.4,.5,.6) | (3,4,5,6) | (.01,.03,.05,.07) | (.03,.04,.05,.06) | 74.20 | 24.90 | 0.75 | 60.22 |
| | 0.4 | (.4,.5,.6,.7) | (4,5,6,7) | (.03,.05,.07,.09) | (.04,.05,.06,.07) | 73.36 | 26.24 | 0.73 | 75.83 |
| 2 | 0.3 | (.3,.4,.5,.6) | (3,4,5,6) | (.01,.03,.05,.07) | (.03,.04,.05,.06) | 87.77 | 34.14 | 0.72 | 89.79 |
| | 0.3 | (.4,.5,.6,.7) | (4,5,6,7) | (.03,.05,.07,.09) | (.04,.05,.06,.07) | 86.88 | 35.60 | 0.70 | 111.99 |
| | 0.4 | (.3,.4,.5,.6) | (3,4,5,6) | (.01,.03,.05,.07) | (.03,.04,.05,.06) | 94.06 | 36.59 | 0.72 | 96.23 |
| | 0.4 | (.4,.5,.6,.7) | (4,5,6,7) | (.03,.05,.07,.09) | (.04,.05,.06,.07) | 93.11 | 38.15 | 0.70 | 120.03 |

9. Concluding Remark

In above table, sensitivity analysis of fuzzy model is presented. The optimum values are presented along with the combination of two values of number of orders (N) and two intervals of values of fuzzy parameters. Decision maker can select the optimum results of any one suitable case. It is expected that if expenditure on advertisement increases, then total inventory cost increases. Also it is seen that, due to increase in the values of various parameters the total inventory cost in fuzzy model increases. So the control on the values of these parameters is necessary. The function principle method is considered in very few fuzzy inventory models for defuzzification. The models are lack of deterioration of an items, so it is necessary to develop the inventory models for deteriorating items with function principle method. Here function principle method is used for defuzzification for shortages-based inventory model of deteriorating items. This model can be extended with finite replenishment. The proposed model can also be developed for multi-item, multi-objective with or without chance constraints.

References:

- 1) Chen, S. H., Wang, C.C., (1996), Backorder Fuzzy Inventory Model under function principle, *Information Sciences*, 95, 71 - 79.
- 2) Chen, S.H., (1985), Fuzzy linear combination of fuzzy linear functions and extension principle and second function principle, *Tamsui Oxford J. Manag. Sci.* 1 (1), 11 - 31.
- 3) Chen, S.H., (1985), Operations on fuzzy numbers with function principle, *Tamkang J. Manag. Sci.* 6 (1), 13 - 26.
- 4) Dubois, T. and Prade, H., (1987), The mean value of a fuzzy number, *Fuzzy Sets and Systems*, 24, 279-300.
- 5) Umap H.P.(2010), Fuzzy EOQ Model for Deteriorating Items with two Warehouses, *Journal of Statistics and Mathematics*, Vol. 1, No.2, pp. 1- 6.
- 6) Umap, H.P., Bajaj V.H. (2007), Fuzzy inventory Model for deteriorating items – Defuzzification by Centroid and Signed distance method, *International Journal of Agricultural and Statistical Sciences*, Vol. 3, No.2, pp. 351-359.
- 7) Vujosevic, M., Petrovic, D. and Petrovic, R., (1996), EOQ formula when inventory cost is fuzzy, *International Journal of Production Economics*, 45, 499 - 504.

- 8) Yao, J.S. and Chiang, J., (2003), Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance, *European Journal of Operational Research*, vol.148, 401 - 409.
- 9) Yao, J.S. and Wu, K.K., (2000), Ranking fuzzy numbers based on decomposition principle and signed distance, *Fuzzy Sets and Systems*, 116, 275 - 288.
- 10) Zadeh, L. A., (1965), Fuzzy sets, *Information and Control*, 8, 338 - 353.
- 11) Zimmerman H.J., (1976), Description and Optimization of Fuzzy System, *International Journal of General Systems*, 2, 209 - 215.